Modeling wave with viscous damping

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Outline

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1. Modeling the propagation of seismic waves
2. Numerical model
3. Exploring the influence of each parameter

Seismic waves in porous rocks

Use of seismic waves to monitor oil, gas reservoir evolution (timelapse seismic).

- Take into account the porous structure of the rocks, partially saturated with fluids (water,oil,gas)
- Observations show frequency dependant attenuations and phase reversal : cannot be explained by conventional acoustic waves theory or poro-elasticity (Biot)

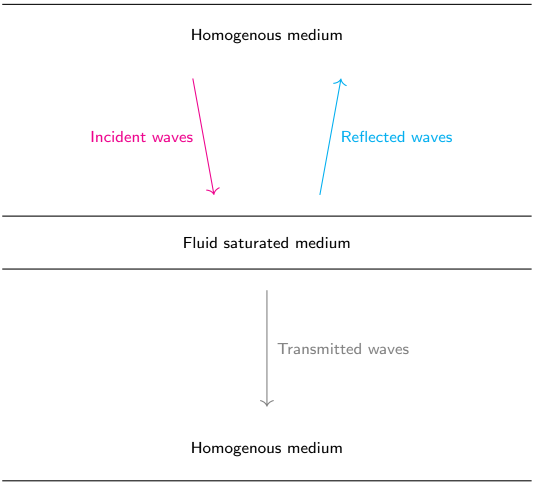
We investigate a model with two additional terms and perfectly matched layers.

[Korneev et al. Geophysics 2001, Goloshubin et al. SEG 2000]

Wave propagation through a fluid saturated medium

When the wave encounters a fluid saturated medium we observe :

- One part is reflected
- One part is transmitted
- The wave amplitude is overall attenuated



Simple acoustic model

The acoustic wave equations reads as :

$$\begin{cases} \dfrac{1}{c(x)^2}\dfrac{\partial^2 u}{\partial t^2}(x,t) - \Delta u(x,t) = f(x,t) \quad \forall x \in \Omega \quad \forall t \in]0,T[, \\ u(x,0) = u_0(x), \quad \dfrac{\partial u}{\partial t}(x,0) = u_1(x) \quad x \in \Omega, \\ +BC. \end{cases}$$

Where $x \mapsto c(x)$ is the propagation speed through the medium, the only adjustable parameter in the model. This model is too simplistic for the case we consider.

Two additional terms for the wave equation

We indroduce two additional terms, responsible for bringing a damping effect to the equation.

A diffusive term $\gamma(x)\frac{\partial u}{\partial t}(x,t)$:

Commonly used. Does not change the nature of the equation.

A viscous term $\eta(x)\frac{\partial}{\partial t}\Delta u(x,t)$

The highest order term.

The diffusive and viscous wave equation

The diffusive and viscous wave equations reads as :

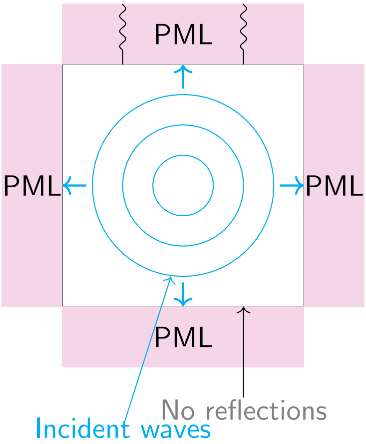
$$\begin{cases} \gamma \dfrac{\partial u}{\partial t}(x,t) - \eta \dfrac{\partial}{\partial t}\Delta u(x,t) + \dfrac{1}{c(x)^2}\dfrac{\partial^2 u}{\partial t^2}(x,t) - \Delta u(x,t) = f(x,t) \\ \forall x \in \Omega \quad \forall t \in]0,T[, \\ u(x,0) = u_0(x), \quad \dfrac{\partial u}{\partial t}(x,0) = u_1(x) \quad x \in \Omega, \\ +BC. \end{cases}$$

Order 3 : The system is no longer hyperbolic.
[Korneev et al. Geophysics 2001, Zhao et al. Geophysics 2014]

Existence theorem : [Han et al. JMAA 2020]

Perfectly Matched Layers

- Simulation in infinite domain: need artificial boundaries.
- Absorbing boundary conditions have not been explored for this model: introduce absorbing layers.
- Perfectly matched layers: no reflection at the interface
- Use PML formulation from [Grote et al. 2010].
  - PML formulation directly from the second order wave equation.
  - This formulation requires fewer auxiliary variables.



The diffusive and viscous equation with PMLs

Extension of the PMLs for the viscous wave equation: [Zhao et al. Geophysics 2022]

$$
\begin{cases}
\dfrac{\partial^2 u}{\partial t^2}(x,t) + (\xi_1 + \xi_2 + \gamma)\dfrac{\partial u}{\partial t}(x,t) + \left[\xi_1\xi_2 + \gamma(\xi_1+\xi_2)\right] u(x,t) \\
+c^2 \Delta u(x,t)) + \eta \nabla \cdot \dfrac{\partial \phi}{\partial t}(x,t) - \gamma \xi_1 \xi_2 \psi(x,t) + c^2 \nabla \cdot \phi(x,t) = f(x,t) \\
\dfrac{\partial \phi}{\partial t}(x,t) = \Gamma_1 \phi(x,t) + \Gamma_2 \nabla u(x,t) \\
\dfrac{\partial \psi}{\partial t}(x,t) = u(x,t)
\end{cases}
$$

$$
\Gamma_1 = \begin{bmatrix} -\xi_1 & 0 \\ 0 & -\xi_2 \end{bmatrix} \text{ et } \Gamma_2 = \begin{bmatrix} \xi_2 - \xi_1 & 0 \\ 0 & \xi_1 - \xi_2 \end{bmatrix}
$$

The functions ξ_1 and ξ_2 characterize the absorption in the layers.

Outline

Discretization

In space : P1 FEM on a triangular mesh.

In time : Implicit centered θ-scheme from Lim et al., Appl. Num. Math., 2007

$$-\Delta(u_h)^n \approx \Delta\left(\theta u_h^{n+1} + (1-2\theta)u_h^n + \theta u_h^{n-1}\right), \qquad \left(\frac{\partial u_h}{\partial t}\right)^n \approx \frac{u_h^{n+1} - u_h^{n-1}}{2\Delta t}.$$

Stable for $\theta \geq \dfrac{1}{4}$.

Discrete Formulation

The discrete system with the PMLs is the following :

$$\int_\Omega \left(\frac{\psi_h^{n+1/2} - \psi^{n-1/2}}{\Delta t} \right) z dx = \int_\Omega u_h^n z dx, \quad z \in V^h(\Omega) \tag{1}$$

$$\begin{aligned}
&\int_\Omega \frac{u^{n+1} - 2u_h^n + u_h^{n-1}}{\Delta t^2} v + \int_\Omega (\xi_1 + \xi_2 + \gamma) \frac{u_h^{n+1} - u_h^{n-1}}{2\Delta t} v dx \\
&+ \int_\Omega [\xi_1 \xi_2 + \gamma(\xi_1 + \xi_2) u_h^n]\, v dx + \eta \int_\Omega \frac{\nabla u_h^{n+1} - \nabla u_h^{n-1}}{2\Delta t} \nabla v dx \\
&+ \int_\Omega c^2 \nabla \left(\theta u_h^{n+1} + (1 - 2\theta) u_h^n + \theta u_h^{n-1} \right) \nabla v dx + \eta \int_\Omega \phi^n \nabla v dx \\
&+ \int_\Omega c^2 \phi^n \nabla(v) dx + \int_\Omega \gamma \xi_1 \xi_2 \frac{\psi^{n+1/2} + \psi^{n-1/2}}{2} v dx = \int_\Omega f^n v dx, \quad v \in V^h(\Omega)
\end{aligned} \tag{2}$$

Discrete Formulation

$$\int_{\Omega} \frac{\phi_1^{n+1} - \phi_1^n}{\Delta t} w = \int_{\Omega} -\xi_1 \phi_1^n w + \int_{\Omega} c^2 \left(\xi_2 - \xi_1\right) \frac{\partial u}{\partial x_1} w dx, \quad w \in V^h(\Omega) \qquad (3)$$

$$\int_{\Omega} \frac{\phi_2^{n+1} - \phi_2^n}{\Delta t} w = \int_{\Omega} -\xi_2 \phi_2^n w + \int_{\Omega} c^2 \left(\xi_1 - \xi_2\right) \frac{\partial u}{\partial x_2} w dx, \quad w \in V^h(\Omega) \qquad (4)$$

The steps must be performed in the indicated order.

The system (1) is explicit and has a reduced computation cost.

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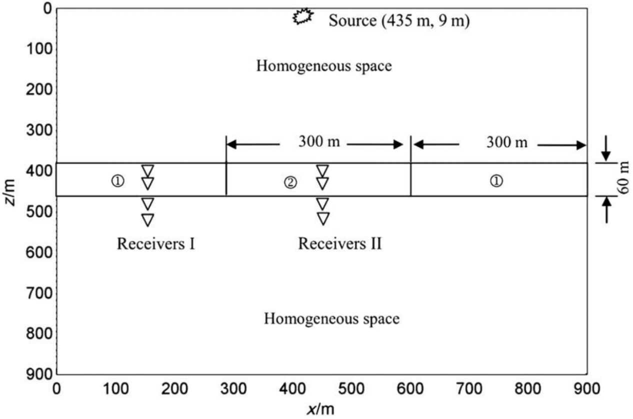
Fluid saturated model containing oil and water



Medium 1 : saturated in oil. Medium 2 : saturated in water
[Zhao et al. 2014 Geophysics]

Characterization of saturated spaces

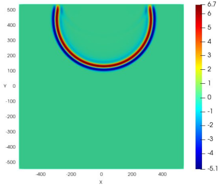
The saturated spaces are characterized by a loss of propagation speed and both damping parameters being non zero.

Table: Initial Damping Parameters

Medium	$\gamma(Hz)$	$\eta(m^2 \cdot s^{-1})$	$c(m \cdot s^{-1})$
Homogenous space	0.0	0.0	1600
Water satured space	90.0	0.02	1470
Oil saturated space	65.4	0.0147	1015

Parameters values from [Zhao et al. 2014 Geophysics].

Acoustic Model



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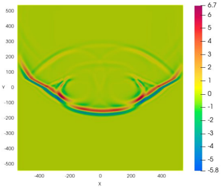

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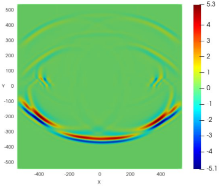

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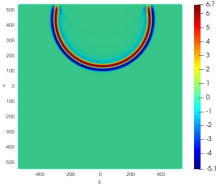
Viscous-diffusive Model



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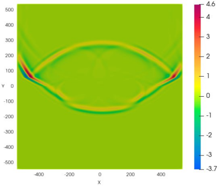

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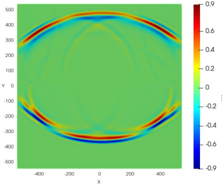

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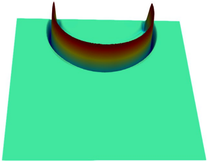
Acoustic Model



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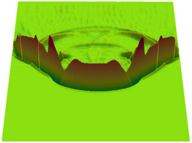

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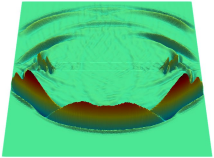

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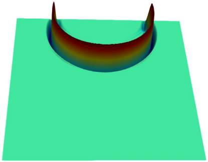
Viscous-diffusive Model



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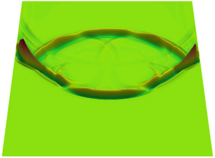

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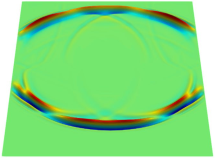

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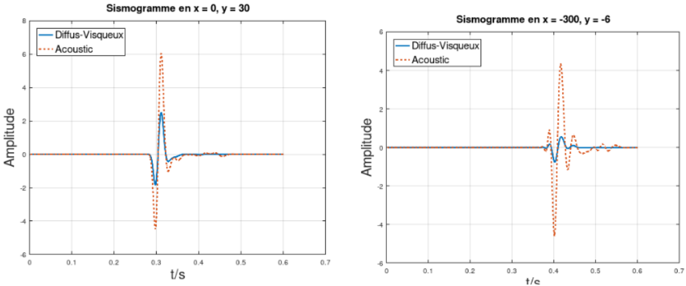
Seismographs

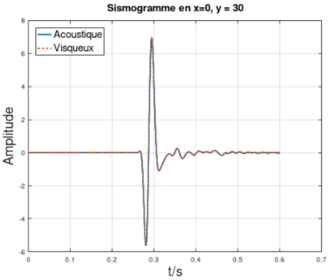


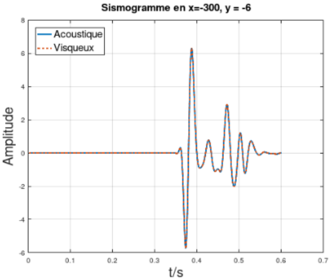
As expected, the wave amplitude is well attenuated while using the viscous-diffusive model...

What is the effect of each damping parameter acting separately?

Effect of the viscous damping parameter

We set $\gamma = 0$ for the damping term $\gamma(x)\frac{\partial u}{\partial t}(x,t)$ and observe the effect of the viscous damping.





No effect!

Harmonic Waves Analysis

We suppose $u = \exp(i\tilde{k}x)\exp(-i\omega t)$ with $\tilde{k} = k + i\alpha$ as a solution of the diffusive-viscous wave equation. α is dimensionless and seen as the attenuation coefficient with the formula

$$\alpha = k_0 \sqrt{\frac{\sqrt{(1-dg)^2 + (d+g)^2} - 1 + dg}{2\,(1+g^2)}}$$

where

$$k_0 = \frac{\omega}{c}, \quad d = \frac{\gamma}{\omega}, \quad g = \frac{\omega\eta}{c^2}.$$

- This yield for $\eta = 0.02$ with $\gamma = 0 \longrightarrow \alpha_\eta = 2.8 \cdot 10^{-9}$
- This yield for $\gamma = 90$ with $\eta = 0 \longrightarrow \alpha_\gamma = 2.1 \cdot 10^{-2}$

$$\alpha_\eta \ll \alpha_\gamma$$

Damping with a similar order of magnitude

Diffusive damping γ

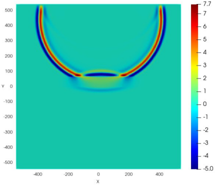
Medium	$\gamma(Hz)$	$\eta(m^2 \cdot s^{-1})$	$c(m \cdot s^{-1})$
Homogenous space	0.0	0.0	1600
Water saturated space	30	0.0	1470
Oil saturated space	21.6	0.0	1015

Viscous damping η

Medium	$\gamma(Hz)$	$\eta(m^2 \cdot s^{-1})$	$c(m \cdot s^{-1})$
Homogenous space	0.0	0.0	1600
Water saturated space	0.0	$1.58 \cdot 10^5$	1470
Oil saturated space	0.0	$1.16 \cdot 10^5$	1015

This yields $\alpha_\eta \approx \alpha_\gamma$

Viscous Model



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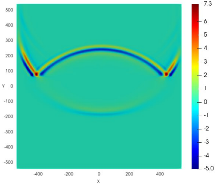

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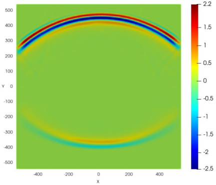

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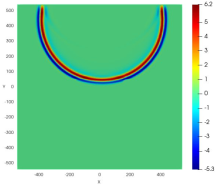
Diffusive Model



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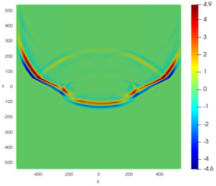

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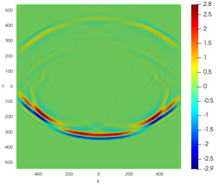

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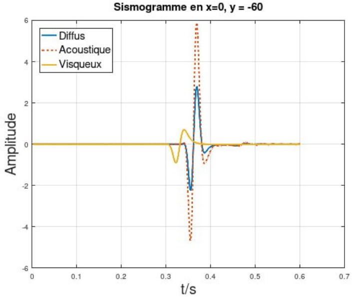

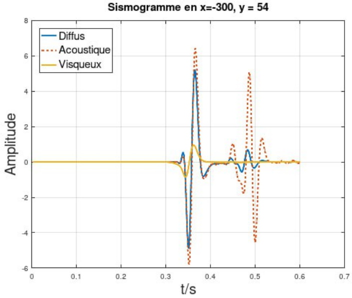

The viscous model produces stronger reflections and a phase reversal effect is observed.

Conclusion

- Accelerate the FreeFem++ code with the use of the parallel module
- Theoretical explanation by computing the reflection/transmission coefficients
- Realistic values for the damping coefficients ?